Some of the most powerful examples of using two-dimensional drawings to represent three-dimensional objects can be found in different types of designing. Architects must be very skilled at drawing three-dimensional objects so there is no confusion about what they represent.

Before a house is built, an architect makes drawings of what the house will look like. The drawings include **elevations** that show how the house will look from two sides.

The plans for a house also include blueprints, which show how the interior will be divided into rooms. Blueprints also show details such as where doors are, which way they open, and where to place items like sinks, the stove, and the bathtub.

**Think About It** Can you think of other careers or companies that use two-dimensional drawings to represent three-dimensional objects?
Dear Student and Family Members,

In Chapter 2, our class will begin studying three-dimensional geometry, including the measurement of surface area and volume. We will build three-dimensional patterns and use words or algebraic expressions to describe them. We will also use blocks to build complex structures that match two-dimensional drawings, as well as make drawings to describe structures we build.

Here is one example of a geometric block pattern that we will build. We will be identifying the pattern and deciding how many cubes there will be in Stage 4, Stage 5, and so on.

![Stage 1, Stage 2, Stage 3]

Another way to draw this pattern is called a *top-count* view. Imagine the view as you are looking down on the block patterns from the top. The first three *top-count* views look like this. The numbers in the drawings show how many blocks are in each stack.

![Stage 1, Stage 2, Stage 3]

Eventually we will try to determine how many blocks are in Stage 93 if there are 8,464 blocks in Stage 92.

**Vocabulary** Along the way, we’ll be learning about these new vocabulary terms:

- base
- prism
- cylinder
- surface area
- net
- volume

**What can you do at home?**

As we near completion of Chapter 2, you and your student might enjoy collecting cans with each of you predicting the volumes of the cans. Then find the actual volume of each can and check to see how close each prediction was to the actual volume. Who was the better predictor?
Using algebra lets you communicate information in a concise way. In this lesson, you will see how you can write algebraic rules to describe geometric patterns made with blocks.

**Explore**

The block pattern below grows from one stage to the next. The square in each stage is larger than the square in the previous stage.

With your own blocks, build Stage 1. Add blocks to build Stage 2. Then add blocks again to build Stage 3.

Continue this pattern. Use your blocks to build Stages 4, 5, and 6.

- When you go from Stage 1 to Stage 2, how many blocks do you add? How do you arrange them?

- When you go from Stage 2 to Stage 3, how many blocks do you add? How do you arrange them?

- What would you do to go from Stage 3 to Stage 4? From Stage 4 to Stage 5? From Stage 5 to Stage 6?

- In general, how many blocks do you add to go from one stage to the next? In other words, how many blocks do you add to Stage \( s \) to make Stage \( s + 1 \)? How do you arrange them?

- How many blocks in total are used in Stage 1? In Stage 2? In Stage 3? In Stage 4? In Stage 5? In Stage 6?

- Write an expression for the number of blocks in Stage \( s \).
Investigation 1 A Staircase Pattern

Here is another block pattern to investigate.

Stage 1

Stage 2

Stage 3

Problem Set A

Build the next three stages of this “staircase” pattern.

1. Describe how you add blocks to go from one stage to the next. That is, tell how many blocks you add, and how you arrange them. Draw pictures if they help to explain your thinking. You may want to use a table like the one below to organize your ideas.

<table>
<thead>
<tr>
<th>To Create Stage</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Add This Many Blocks</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Marty thinks there are 37,401 blocks in Stage 273. If he’s right, how many blocks are in Stage 275?

3. How many blocks do you add to Stage $s$ to make Stage $s + 1$? Write an expression to describe the number of blocks you add, and tell how you would arrange them.

Look at the number of blocks in each stage. If you made a table for Problem 1, you may want to add a row to your table to help you find a pattern to solve the next few problems.

<table>
<thead>
<tr>
<th>To Create Stage</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Add This Many Blocks</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Number of Blocks Needed</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. How many blocks does it take to build Stage 10?

5. How many blocks does it take to build Stage 100?

6. Write an expression to describe the number of blocks you need to build Stage $s$. 
7. **Prove It!** Find a way to explain why the expression you wrote for Problem 6 is correct, based on how you built each stage of the staircase pattern. To help you get started, you might try these ideas:

- Think about building two copies of one staircase and putting them together.
- Try breaking up a staircase into smaller pieces.
- Rewrite your expression in a different way, and try to fit both expressions to how the staircase grows.

**Investigation 2 Other Block Patterns**

Here is another block pattern to build and investigate.

Another way to represent this “double staircase” pattern is to use a *top-count view*. Imagine you are looking down on the buildings from above. The numbers in the drawings below show how many blocks are in each stack.
Problem Set B

1. Build the next three stages of this pattern. Draw a top-count view of each stage.

2. Think about how you go from one stage to the next.
   a. How many blocks do you add to Stage 1 to make Stage 2? How do you arrange them?
   b. How do you add blocks to Stage 2 to make Stage 3?
   c. How do you add blocks to Stage 3 to make Stage 4?

3. If Stage 92 has 8,464 blocks, how many blocks are in Stage 93?

4. How many blocks do you add to Stage \( s \) to make Stage \( s + 1 \)? Write an expression to describe the number of blocks you add, and tell how you would arrange them.

5. How many blocks does it take to build Stage 3? Stage 4? Stage 5?

6. How many blocks would it take to build Stage 25? Stage 1,000?

7. There are 100 blocks in a particular stage of this pattern. Which stage is it?

8. If you have 79 blocks, which is the largest stage of the pattern you could build?

9. Write an expression to describe how many blocks you need to build Stage \( s \). Explain why your expression works.

Think & Discuss

Suppose you were making a pattern of larger and larger cubes. Stage 1 would look like this:

Stage 1

Which of these figures is Stage 2 in a growing pattern of cubes?

A  B  C

Remember

A cube is a three-dimensional figure with six square sides, or faces.
Problem Set C


2. How many blocks did you use to build Stage 2? Stage 3? Stage 4?

3. Without building it, predict how many blocks you would need to build Stage 5.

4. Write an expression to describe how many blocks you need to build Stage \( s \). Explain how you found your answer.

Share & Summarize

Compare all the block patterns you have seen. Thinking only about the numbers of blocks needed to build each pattern, which of the patterns grow more quickly? Which grow more slowly? Do any grow in the same way, using the same numbers of blocks at each stage?

Investigation 3 Building Block Patterns

Zoe noticed that the square pattern and the double-staircase pattern grow the same way numerically.

Think & Discuss

The cartoon on the next page shows how Zoe thought about creating the square pattern from the staircase pattern. Can you work the other way? That is, can you find a way to rearrange the blocks from the squares to form the double staircase? Describe your method.
Problem Set

1. Work with your group to build a third block pattern that grows according to the same numerical pattern as the squares and the double staircase. Stage 1 will have 1 block, Stage 2 will have 4 blocks, and Stage \( s \) will have \( s^2 \) blocks. Draw the first three stages of your pattern using top-count views.

2. Now build another pattern that grows like the staircase pattern from Investigation 1: Stage 1 will have 1 block, Stage 2 will have \( 1 + 2 \) blocks, Stage 3 will have \( 1 + 2 + 3 \) blocks, and so on. Draw the first three stages of your pattern using any method you like.

3. Build a pattern with \( s^3 - 1 \) blocks in Stage \( s \). Stage 1 will have no blocks, Stage 2 will have 7 blocks, Stage 3 will have 26 blocks, and so on.

Share & Summarize

Create your own block pattern.

1. Draw at least the first three stages of your pattern.

2. Describe how your pattern grows from one stage to the next.

3. Write an expression for the number of blocks in each stage of your pattern.
On Your Own Exercises

1. Squares can be used to make growing patterns. Here is a growing pattern of L-shapes.
   
   ![L-shapes diagram](image)
   
   a. How many squares are added to Stage 1 to make Stage 2? To Stage 2 to make Stage 3?
   b. How many squares do you add to go from Stage $s$ to Stage $s + 1$?
   c. How many squares are used in Stage 1? In Stage 2? In Stage 3?
   d. Write an expression for the number of squares in Stage $s$.

2. Here is a growing pattern of H-shapes.
   
   ![H-shapes diagram](image)
   
   a. How many squares are added to Stage 1 to make Stage 2? To Stage 2 to make Stage 3?
   b. How many squares do you add to go from Stage $s$ to Stage $s + 1$?
   c. How many squares are used in Stage 1? In Stage 2? In Stage 3?
   d. Write an expression for the number of squares in Stage $s$.

3. Choose a letter of the alphabet that you can draw with squares. Some good choices are A, C, E, F, I, O, T, U, and X.
   
   a. Draw Stage 1, Stage 2, and Stage 3 versions of your letter. Make sure the letter grows predictably from one stage to the next.
   b. How many squares are added from Stage 1 to Stage 2? From Stage 2 to Stage 3?
   c. How many squares do you add to go from Stage $s$ to Stage $s + 1$?
d. How many squares are in Stage 1? In Stage 2? In Stage 3?
e. Write an expression for the number of squares in Stage \( s \) of your letter.

4. Here is a new block pattern.

The top-count views of Stages 1 and 2 are shown at left.

a. Draw the top-count views of Stages 3 and 4.
b. How many blocks are added to Stage 1 to make Stage 2? To Stage 2 to make Stage 3?
c. How many blocks do you add to go from Stage \( s \) to Stage \( s + 1 \)?
d. How many blocks are used in Stage 1? In Stage 2? In Stage 3?
e. Write an expression for the number of blocks in Stage \( s \).

5. Look at the block pattern in Exercise 4.

a. Describe how you could modify the pattern so that only 4 blocks are added from one stage to the next.
b. Use any method you like to draw the first three stages of your pattern.

6. Here is another block pattern.

a. How many blocks are added to Stage 1 to make Stage 2? To Stage 2 to make Stage 3?
b. How many blocks are added to get from Stage \( s \) to Stage \( s + 1 \)?
c. How many blocks are used to make Stage 1? Stage 2? Stage 3?
d. Write an expression for the number of blocks in Stage \( s \).

Remember
To be a pattern, blocks must be added in a predictable way.
7. Draw the first three stages of another pattern that uses the same number of blocks in each stage as the pattern in Exercise 6. You may want to use top-count views.

8. **Challenge** To build Stage 2 of this block pattern, you could make its base (the bottom layer) and set Stage 1 on top of it. To build Stage 3, you could make its base and set Stage 2 on top of it.

- **Stage 1**
- **Stage 2**
- **Stage 3**

**a.** There are 5 blocks in the base of Stage 2. How many blocks are in the base of Stage 3?

**b.** How many blocks are in the base of Stage \( s \)?

**c.** How many blocks are used in Stage 1? In Stage 2? In Stage 3?

**d.** Describe a rule for the number of blocks in Stage \( s \). Write an expression for the rule, or describe it in words.

9. Jamie found that the number of blocks in Stage \( s \) of his block pattern was \( 2s \).

- **a.** Copy and complete this table for Jamie’s pattern.

<table>
<thead>
<tr>
<th>Stage</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blocks</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- **b.** What might Jamie’s block pattern look like? Draw the first three stages.

- **c.** How many blocks are used in Stage 10 of Jamie’s pattern?

- **d.** How many blocks does Jamie add to Stage 1 to make Stage 2? To Stage 2 to make Stage 3?

- **e.** How many blocks does Jamie add to one stage to build the next stage in his pattern?
10. Kate found that the number of blocks in Stage $s$ of her block pattern was $s^2 - 1$.

a. Copy and complete this table for Kate’s pattern.

<table>
<thead>
<tr>
<th>Stage</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blocks</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. What might Kate’s block pattern look like? Draw the first three stages.

c. How many blocks are used in Stage 10 of Kate’s pattern?

d. How many blocks does Kate add to Stage 1 to make Stage 2? To Stage 2 to make Stage 3?

e. Describe how many blocks Kate adds to one stage to build the next.

11. Keisha drew a growing line pattern on dot paper.

Remember
To find the area of a figure on dot paper, you can count the squares inside the figure. Sometimes you have to count half squares.

a. Make a table showing the stage, the number of dots on the perimeter of the figure, and the area of the figure.

<table>
<thead>
<tr>
<th>Stage</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dots on Perimeter</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Area (square units)</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. Write an expression for the number of dots on the perimeter in Stage $s$.

c. Write an expression for the area of Stage $s$. 
12. Keisha drew the shrinking line pattern shown at left on dot paper.
   
a. Make a table showing the stage, the number of dots on the perimeter of the figure, and the area of the figure.

<table>
<thead>
<tr>
<th>Stage</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dots on Perimeter</td>
<td>24</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Area (square units)</td>
<td>36</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. Write an expression for the number of dots on the perimeter in Stage $s$.

c. Write an expression for the area of Stage $s$, or describe in words the area that is left.

d. At what stage will the figure have area 0? Does it make sense to continue the pattern beyond that stage?

13. Every morning, Minowa helps raise the flag up the school’s flagpole. She hooks the flag onto the rope at a height of 3 ft. She found that with each pull on the rope, the flag rises 2 ft.

   a. How high is the flag after three pulls?

   b. Write an expression to describe the flag’s height after $p$ pulls.

   c. If the flagpole is 40 ft high, how many pulls does it take Minowa to get the flag to the top?

14. At a popular clothing store, clothes go on sale when they have hung on the rack too long. When an item is first put on sale, the store marks the price down 10%. Every week after that, the store takes an additional 10% off the original price. So one week after the first markdown, you get 20% off; after 2 weeks, you get 30% off; and so on.

   a. If some shoes are regularly priced at $50, how much will they cost after the first discount?

   b. How much will the shoes cost 2 weeks after the first discount?

   c. When will the shoes be half price?

   d. When will the shoes cost $30?

   e. Write an expression for the price of the shoes $w$ weeks after the first discount is taken.

   f. When an item is discounted to 100% off, the store donates it to charity. If the shoes don’t sell, how long will they stay in the store before they are donated? Does your answer depend on the original price of the item?
15. Joaquin owes his sister $30 for taking him to a concert. He earns $5 each week for doing chores. He promised to give his sister half his earnings every week until the $30 is paid.
   a. How much does Joaquin owe his sister after 1 week?
   b. Write an expression for how many dollars Joaquin owes his sister after \( w \) weeks.
   c. How many weeks will it take Joaquin to pay off his debt?

16. Preview Biologists take water samples from a particular swamp in the beginning of spring. They estimate there are 100 mosquito larvae per square meter in the swamp. The area has had a very rainy spring, so the mosquito population is growing quickly, tripling every month.
   a. How many mosquito larvae are there per square meter 1 month after the scientists first measured? 2 months after?
   b. If there are \( x \) larvae in a given month, how many are there in the next month? In the month after that?
   c. How can you find the number of mosquito larvae in a square meter \( m \) months after the scientists first measured?
   d. After how many months will there be more than 5,000 mosquito larvae per square meter?
   e. Would this growth continue forever? Explain your answer.

17. Here is a shrinking pattern that starts with a square sheet of paper. Each stage is the same height as, and half as wide as, the previous stage.

   a. What happens to the area of the paper from one stage to the next?
   b. What happens to the perimeter of the paper from one stage to the next?
   c. Will the paper ever disappear (have an area of 0 units)? Explain your answer.
18. **Challenge** Malik found that the number of blocks in Stage \( s \) of his block pattern is \( s^3 + 3 \).

- **a.** How many blocks are in Stage 10 of Malik’s pattern?
- **b.** How many blocks does Malik add to go from Stage 1 to Stage 2? From Stage 2 to Stage 3? From Stage 3 to Stage 4?
- **c.** Without calculating the number of blocks in each stage, predict how many blocks Malik will add to go from Stage 4 to 5 and from Stage 5 to 6. Explain how you found your answer.

**Mixed Review**

Rewrite each expression using exponents.

- **19.** \( t \times t \times t \)
- **20.** \( \pi \times r \times r \)
- **21.** \( y \times y \times y \times y \times y \)

Order each group of numbers from least to greatest.

- **22.** \( \frac{1}{3}, \frac{2}{3}, 1, \frac{5}{3}, 0 \)
- **23.** \( \frac{2}{6}, \frac{1}{6}, -\frac{1}{2}, -1, -\frac{3}{6}, 1 \)
- **24.** 4, 0, 0.4, -0.04, -\( \frac{1}{4}, \frac{1}{4} \)
- **25.** \( \frac{3}{8}, 0.125, \frac{3}{8}, \frac{9}{8}, 0 \)
- **26.** 3, 30, 0.3, 0.03, \( \frac{3}{10} \)
- **27.** \(-1, 10, -0.1, \frac{1}{5}, 0, 0.005 \)

Fill in the blanks to make true statements.

- **28.** \( \frac{1}{8} + \_ = 1 \)
- **29.** \( \frac{8}{5} - \_ = 1 \)
- **30.** 0.5 + \_ = 1
- **31.** 0.5 + 0.25 + \_ = 1
- **32.** \( \frac{9}{12} + \frac{3}{12} + \_ = 1 \)
- **33.** \( \frac{3}{10} + \frac{7}{10} - \_ = 1 \)

Find the product or quotient.

- **34.** 156 \( \times \) 3
- **35.** 12 \( \times \) 25
- **36.** 52 \( \div \) 13
- **37.** 352 \( \div \) 11

38. Kiran wants to display his collection of 30 first-class stamps in groups of equal size. What group sizes are possible?
Visualizing and Measuring Block Structures

One way to describe a structure is to draw how it looks from different viewpoints. For example, here are six views that might describe the block structure at left.

These flat views are like shadows that would be cast against a wall or a floor by the structure, but they also show the lines between the blocks. The top view is a “bird’s-eye view,” looking down from above. For the bottom view, imagine building the structure on a glass table and looking at it from underneath. The left and right views show how the structure looks when viewed from those sides.

Explore

Build this block structure. Then draw the six views of it using graph paper or dot paper. If you build your structure on a sheet of paper, you can write the labels Front, Back, Left, and Right on the paper. This will make it easier for you to remember which side is which.

Which of your views look exactly the same?
Investigation 1 Seeing All the Angles

In this lesson, you will consider only block structures in which at least one face of each block matches up with a face of another block. (Creating views like those in the Explore on page 91 can be difficult for other kinds of buildings.) Keep this in mind as you build your structures.

Since you will build these structures on your desk, the laws of gravity apply. Blocks need to rest on the table or on other blocks, so you can’t make a structure like the one at left.

**Problem Set A**

Work with your partner to build three different block structures. Use 6 to 10 blocks for each of them, and follow the rules of good structures. Keep the structures in front of you for this set of problems.

1. First, focus on top and bottom views.
   a. For each of your structures, draw top and bottom views. Be sure to label which is which.
   b. Describe in words the relationship between the top and bottom views of a structure.
   c. Does that relationship hold for any block structure, or just for those you built?

2. Now focus on left and right views.
   a. For each structure, draw and label left and right views.
   b. Describe the relationship between the left and right views of a structure.
   c. Does that relationship hold for any block structure?

3. Finally, focus on front and back views.
   a. For each structure, draw and label front and back views.
   b. Describe the relationship between the front and back views of a structure.
   c. Does that relationship hold for any block structure?
Problem Set B

Use the relationships you discovered in Problem Set A to complete these problems.

1. This is the right view of a structure. Draw the left view.

   ![Left View Diagram]

2. This is the top view of a structure. Draw the bottom view.

   ![Bottom View Diagram]

3. This is the front view of a structure. Draw the back view.

   ![Back View Diagram]

Share & Summarize

1. Suppose you know the top view of a structure. Would you learn anything more about the structure from the bottom view? Explain your answer.

2. If you know the front, top, and right views of a structure, would any other view give you more information about the structure? Explain.
Investigation 2 Different Views

Think & Discuss

In many problems in this chapter, you have used top-count views to describe block structures. For this top-count view, is there more than one possible structure?

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Is there any top-count view that could fit more than one structure? Explain your answer.

Problem Set C

You will now explore this question: Can more than one structure fit the same front, top, and right views?

1. Working on your own, use 10 to 15 blocks to build a block structure.
   a. Draw a top-count view for your structure.
   b. On another sheet of paper, draw front, top, and right views. Then take your structure apart.
   c. Exchange front, top, and right views with a partner, and build a structure that matches your partner’s drawings. Did you build the structure your partner had in mind? (Use the top-count view to check.)

2. Here are three views of a block structure.
   - a. Build a structure that fits these views. Draw a top-count view for your structure.
   - b. Build a different structure that fits these views, and draw a top-count view.
   - c. Do you think there are other possible structures? Why or why not?
Creating a structure from three given views is an interesting puzzle, but if three views represent lots of different structures, they are not very useful. If you were designing a house, for instance, you would want to be very specific in your drawings so that what you imagine is what gets built.

Engineers and architects often need to make drawings of structures, but the types of drawings they make must give more precise information about a structure. The drawings you will work with next can give more information about the block structures they represent.

Just the facts

Although top-count views show exactly how the blocks are arranged in a block structure, they aren't really useful to engineers and architects because things in real life aren't usually made of blocks.

EXAMPLE

Remember this structure?

Below is another type of three-view drawing that describes it. This type of drawing shows the different levels of the block structure but not the lines between blocks.

The line in the front view shows that there are two different levels of blocks in the structure. In this structure, the base extends beyond the tall part.

The line in the top view shows that there are two different heights of blocks in the structure. The right view shows that the part in front is lower.

These three views together often contain more information than the front, top, and right views you have used before. We will refer to the earlier kind of views as regular views and the new kind as engineering views.

It is sometimes hard to picture a structure from just looking at the engineering views. Practice will help!
Problem Set D

Build the structure described by each set of three views. Draw a top-count view to record each structure you build.

1. [Front] [Top] [Right]

2. [Front] [Top] [Right]

3. [Front] [Top] [Right]

Share & Summarize

1. Can there be more than one structure with the same regular front, top, and right views? Explain your answer by giving an example.

2. The three structures in Problem Set D all fit the same regular front, top, and right views. How do the engineering views show the differences among the structures?
In Problem Set D, you learned to interpret engineering views and to show the structures they represent with top-count views. Now you will draw engineering views yourself.

**Problem Set E**

Work with a partner to draw the three engineering views for each structure shown. It will probably help if you build the structures.

1. 
2. 
3. 
4. 
5. 
6. 

7. Working on your own, use 10 to 15 blocks to build a block structure.
   a. Draw a top-count view for your structure.
   b. On another sheet of paper, draw front, top, and right engineering views. Then take your structure apart.
   c. Exchange engineering views with a partner, and build a structure that matches your partner’s drawings. Did you build the structure your partner had in mind? (Use the top-count view to check.)

**Share & Summarize**

1. Use 6 to 10 blocks to build a new structure. Then draw each of the following:
   - the top-count view
   - the regular front, top, and right views
   - the engineering front, top, and right views

2. Which do you find easier to work with: regular front, top, and right views; engineering views; or top-count views? Why?
Investigation 4  Measuring Block Structures

Area, which is the space inside a two-dimensional figure, is measured in square units. To find the area of an irregular figure like the one shown here, you can count squares (shown with dashed lines).

You can say that the area of this figure is 8 square units. Or, if you know the size of the squares, you can use it to state the area exactly. For example, these squares have a side length of 1 centimeter, so the area of the figure is 8 square centimeters. This can be written 8 cm².

The surface area of a three-dimensional object is the space covering the object’s surface. If you could open up the object and flatten it so you could see all sides at once, the area of the flat figure would be the surface area. (Don’t forget to count the bottom surface!) Surface area is also measured in square units.

Volume, the space inside a three-dimensional object, is measured in cubic units. If the blocks you build with are each 1 cubic unit, then the volume of a block structure is equal to the number of blocks in the structure. For example, a structure made from eight blocks has a volume of 8 cubic units. If the blocks have an edge length of 1 cm, the structure’s volume is 8 cm³.

In the problems that follow, the blocks each have an edge length of 1 unit, faces of area 1 square unit, and a volume of 1 cubic unit.

Think & Discuss

What is the surface area of a single block in square units?

If the edge lengths of a block are 2 cm, what is the block’s surface area?

What is the volume of the structure at the right in cubic units?

What is the surface area of the structure above in square units? (Remember: Count only the squares on the outside of the structure.)
LESSON 2.2 Visualizing and Measuring Block Structures

**Materials**
- cubes
- graph paper or dot paper

**Problem Set F**

1. Find the volume and the surface area of each three-block structure.
   
   a. 
   b. 

2. Find the volume and the surface area of each four-block structure.
   
   a. 
   b. 
   c. 
   d. 
   e. 
   f. 

3. Do the structures in Problem 2 all have the same volume? Explain your answer.

4. Which of the structures in Problem 2 have the greatest surface area? Which has the least surface area?

5. Build two block structures with at least six blocks each that have the same volume but different surface areas.
   
   a. For each structure, draw a top-count view.
   b. Record the volume and the surface area of each structure.

**Share & Summarize**

Make a block structure with the same volume as this structure but with less surface area. How did you decide how to rearrange the blocks?
On Your Own Exercises

1. Here is a block structure and four views. Some of the views match the structure, but at least one does not. The top-count view is given to help you know how the blocks are arranged. For each view, write top, front, right, or not possible.

   ![Block Structure and Views](image)

   a. b. c. d.

2. Here is a block structure and five views. Some of the views match the structure, but at least one does not. The top-count view is given to help you know how the blocks are arranged. For each view, write top, front, right, or not possible.

   ![Block Structure and Views](image)

   a. b. c. d. e.
3. Here is Felipe’s block structure.

4. Here are three views for a structure.

Choose which of the following structures fit these views. Does more than one fit? The top-count views are given to help you know how the blocks are arranged.

a. 

b. 

c. 

5. Here is a block structure and four engineering views. Some of the views match the structure, but at least one does not. For each engineering view, write top, front, right, or not possible.

```
  2 1 1
  2 1 1
  2 2 2
```

a. b. c. d.

6. Here is Maya’s block structure.

a. Draw an engineering top view of Maya’s structure.
b. Draw an engineering front view of Maya’s structure.
c. Draw an engineering right view of Maya’s structure.

7. Here are three engineering views for a structure. Choose which of the three structures shown fit these views. Does more than one fit?

```
Front Top Right
```

a.
b.

\[
\begin{array}{ccc}
1 & 1 & 1 \\
1 & 1 & 1 \\
2 & 1 & 2 \\
\end{array}
\]

c.

\[
\begin{array}{ccc}
2 & 1 & 2 \\
1 & 1 & 1 \\
2 & 1 & 2 \\
\end{array}
\]

Find the volume and the surface area of each structure.

8.

\[
\begin{array}{ccc}
1 & 1 & 1 \\
1 & 2 & 2 \\
1 & 2 \\
\end{array}
\]

9.

\[
\begin{array}{ccc}
3 & 2 & 1 \\
2 & 2 & 1 \\
1 & 1 & 1 \\
\end{array}
\]

10.

\[
\begin{array}{ccc}
2 & 1 & 1 \\
2 & 1 & 1 \\
2 & 2 & 2 \\
\end{array}
\]

11.

\[
\begin{array}{ccc}
2 & 1 & 2 \\
1 & 1 & 1 \\
2 & 1 & 2 \\
\end{array}
\]

Remember

*Volume* is measured in cubic units; *surface area* is measured in square units.
Below are top-count views for four block structures. Find the volume and the surface area of each structure.

12. 13.

14. 15.

16. Draw top-count views for three structures that each have a volume of 9 cubic units. Find the surface area of each structure.

17. If possible, draw a top view for a block structure so that the bottom view would be identical. If you have blocks available, you may want to build the structure.

18. If possible, draw a right view for a block structure so that the left view would not be identical. If you have blocks available, you may want to build the structure.

19. If possible, draw a front view of a block structure so that the back view would show a different number of squares.

*Polyominoes* are figures that are formed by joining squares with edges lined up exactly.

Two polyominoes are the same if you can cut them out of the paper and fit them exactly on top of each other.
20. Which of these are polyominoes?

a. 
```
  
```
b. 
```
  
```
c. 
```
  
```
d. 
```
  
```
e. 
```
  
```

21. Which of the polyominoes below are the same?

a. 
```
  
```
 b. 
```
  
```
c. 
```
  
```
d. 
```
  
```
e. 
```
  
```
f. 
```
  
```
g. 
```
  
```
h. 
```
  
```
i. 
```
  
```

22. You have probably played the game of dominoes. In the game, dominoes are tiles that have two squares containing dots. In mathematics, *dominoes* are just polyominoes made of two squares. Considering the *mathematical* definition, how many different dominoes are there? Draw them.

23. *Trominoes* are built with three squares. How many different trominoes are there? Draw them.

24. A popular computer game is played with tetrominoes (polyominoes with four squares). These pieces are used:

As the pieces fall from the top of the screen, you can move them left or right and rotate them clockwise and counterclockwise, trying to create solid rows (with no gaps) when the pieces land near the bottom of the screen.

a. Are all the pieces different tetrominoes, or are some of them the same?

b. Why must this game include more than one piece to represent the same tetromino?

25. *Preview* Hexominoes are built with six squares.

   a. Draw at least six different hexominoes.

   b. You can cut out some hexominoes along the outside and fold them along the lines on the inside to make a cube.

   Why is it useful to be able to build structures from flat views? In what kinds of jobs would visualizing be an important skill?
26. You can draw three engineering views for objects other than block structures. For example, here are front, top, and right views for a roll of tape.

![Front Top Right](image)

Draw front, top, and right engineering views for two other common objects. Name each object.

27. Draw front, top, and right views of a sphere. It might help to look at a ball.

![Sphere](image)

28. Some engineers draw mechanical devices, like gears. Draw front, top, and right engineering views for this simple gear.

![Gear](image)

**Preview** Suppose you had a different kind of block to build with, a half cube. You could build structures like those below. Find the volume, in cubic units, of each structure.

29.  
30.  
31.  

32. Estimate the volume of a room in your home in 1-inch blocks. Describe your method.

Find the indicated percentage.

33. 50% of 12  
34. 12% of 50  
35. 25% of 12  
36. 12% of 25  
37. 75% of 12  
38. 12% of 75  

39. Which of the following are factors of 36?

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>24</th>
<th>36</th>
</tr>
</thead>
</table>

**Mixed Review**
The number pair “2, 10” is a factor pair of 20 because $2 \times 10 = 20$. For each number below, list all the factor pairs. (The factor pairs “2, 10” and “10, 2” are the same.)

40. 5
41. 10
42. 25
43. 18

Evaluate each expression.

44. $\frac{3}{7} - \frac{2}{7}$
45. $\frac{8}{7} - 1$
46. $\frac{3}{8} + \frac{4}{8} - \frac{1}{8}$
47. $\frac{1}{2} \times \frac{9}{3}$
48. $\frac{1}{8} \times \frac{3}{5}$
49. $\frac{4}{7} \times \frac{1}{4}$
50. $\frac{3}{5} \div \frac{1}{2}$
51. $\frac{1}{4} \div \frac{3}{4}$
52. $\frac{1}{8} \div \frac{3}{8}$

Sketch a graph to match each story.

53. Paul charted his height from when he was 3 until he turned 12.
54. The space shuttle was launched on Monday, orbited the planet for 10 days, and then returned to Earth.
55. Batai threw the ball into the air and watched as it rose, fell, and then hit the ground.

56. Write each of the numbers 1 through 35 in a diagram like the one below. If a number does not fit in any of the circles, write it outside the diagram.
These figures are prisms.

These figures are not prisms.

Think & Discuss

What do all the prisms have in common?

How are the nonprisms different from the prisms?

All prisms have two identical, parallel faces. These two faces are always polygons. A prism’s other faces are always parallelograms.

A prism is sometimes referred to by the shape of the two identical faces on its ends. For example, a triangular prism has triangular faces on its ends, and a rectangular prism has rectangular faces on its ends.
Investigation 1

Finding Volumes of Block Structures

If a block structure has a constant height—that is, has the same number of blocks in every column—that structure is a prism. If the top view of such a structure is a rectangle, the structure is a rectangular prism.

Problem Set A

By using its dimensions, you can describe a rectangular prism exactly. For example, a prism with edge lengths 3 units, 2 units, and 4 units is a $3 \times 2 \times 4$ prism.

1. Make all the rectangular prisms you can that contain 8 blocks.
   - a. Record the dimensions, volume, and surface area of each prism you make.
   - b. Do the 8-block rectangular prisms all have the same volume?
   - c. Which of the 8-block rectangular prisms has the greatest surface area? Give its dimensions.
   - d. Which of the prisms has the least surface area?

2. Make all the rectangular prisms you can that have a volume of 12 cubic units.
   - a. Record the dimensions and surface area of each prism you make.
   - b. Which prism has the greatest surface area?
   - c. Which prism has the least surface area?

3. Now find all the rectangular prisms that have a volume of 20 cubic units. Try to do it without using your blocks.
   - a. Record the dimensions and surface area of each prism.
   - b. Which prism has the greatest surface area?
   - c. Which prism has the least surface area?
Problem Set B

1. Here is a top view of a prism.

   a. Build a prism 1 unit high with this top view. What is its volume?
   b. Build a prism 2 units high with this top view. What is its volume?
   c. What would be the volume of a prism 10 units high with this top view?
   d. Write an expression for the volume of a prism with this top view and height \( h \).
   e. What is the area of this top view?

2. Here is another top view.

   a. Build a prism 1 unit high with this top view. What is its volume?
   b. Build a prism 3 units high with this top view. What is its volume?
   c. Suppose you built a prism 25 units high with this top view. What would its volume be?
   d. Write an expression for the prism’s volume, using \( h \) for height.
   e. What is the area of this top view?

3. Here is a third top view.

   a. Build a prism 1 unit high with this top view. What is its volume?
   b. Build a prism 5 units high with this top view. What is its volume?
   c. Suppose you cut your blocks in half to build a structure half a unit high with this top view. What would its volume be?
   d. Write an expression for the prism’s volume, using \( h \) for height.
   e. What is the area of this top view?

Remember
Your block has an edge length of 1 unit and a volume of 1 cubic unit. Each face has an area of 1 square unit.
Investigation 2

Finding Other Volumes

Will your method for finding the volume of a prism work for prisms that are not block structures? The problems that follow will help you find out.

Remember that a prism has two identical, parallel faces that can be any type of polygon. These faces are called the bases of the prism.

Problem Set C

1. This triangle is half of the face of one of your blocks.

   a. What is the area of this triangle?

   b. If you cut one of your blocks in half as shown here, you could build a structure 1 unit high with the triangle as its base. What would the volume of this structure be?

   c. If you built a structure 10 units high that had the triangle as its base, what would its volume be?
2. The dashed lines on the parallelogram below show the relationship between the parallelogram and a face of one of your blocks.

   ![Parallelogram](image)

   a. What is the area of the parallelogram?
   b. If you had blocks like the one shown here, you could build a structure 2 units high that had the parallelogram as its base. What would its volume be?
   c. If you could build a structure $h$ units high with this parallelogram as its base, what would its volume be?

VOCABULARY

cylinder

A cylinder is like a prism, but its two bases are circles. These are all cylinders.

3. The dashed lines on the circle below show the relationship between the circle and a face of one of your blocks.

   ![Circle](image)

   a. What is the area of the circle?
   b. If you had blocks like this one, you could build a structure 3 units high with the circle as its base. What would its volume be?
   c. If you could build a structure 1,000 units high with this base, what would its volume be?

Remember

The area of a circle is $\pi r^2$, where $r$ is the radius and $\pi$ is about 3.14.
4. Luis drew a floor plan for a playroom.

\[
\begin{array}{c}
10 \text{ ft} \\
12 \text{ ft}
\end{array}
\]

a. What is the area of the floor?
b. If the playroom has 8-ft ceilings, what is its volume?
c. If the playroom has 10-ft ceilings, what is its volume?

5. Here is a design for a large oil tank.

\[
\begin{array}{c}
9 \text{ ft}
\end{array}
\]

a. What is the area of the base of the tank?
b. If the tank is 50 ft high, how much water will it hold (in cubic feet)?
c. If the tank is \( h \) ft high, how much water will it hold?

Think & Discuss

Look back over your answers for Problem Set C.

- Describe a single strategy that you think will work for finding the volume of any prism. Test your strategy with the prisms in Problem Set C.

- Will the same strategy work for the volume of any cylinder? If not, can you modify the strategy so that it will work?
Problem Set D

Some prisms, called *oblique prisms*, are slanted. However, you have only been finding the volumes of prisms with sides that are straight up and down, which are called *right prisms*.

1. This right prism has been sliced into very thin pieces, like a deck of cards. These “cards” have then been pushed into an oblique prism, but the cards themselves haven’t changed.

   {image of right and oblique prisms}

   a. If each card is 5 cm wide and 8 cm long, and the stack is 15 cm high, what is the volume of the first “deck”?

   b. Is the volume of the second deck the same as, greater than, or less than the volume of the first deck? Explain your answer.

   c. Is the base of the second deck the same as, larger than, or smaller than the base of the first deck? Explain.

   d. The diagram at left shows how to measure the height of an oblique prism. Is the height of the second deck the same as, greater than, or less than the height of the first deck? Explain.

2. Now think about a deck of circular cards.

   {image of circular cards}

   a. If the cards each have radius 4 cm, and the stack is 9 cm high, what is the volume of the first deck?

   b. Is the volume of the second deck the same as, greater than, or less than the volume of the first deck? Explain.

   c. Is the base of the second deck the same as, larger than, or smaller than the base of the first deck? Explain.

   d. Is the height of the second deck the same as, greater than, or less than the height of the first deck? Explain.

3. Will the formula \( \text{area of base} \times \text{height} \) give you the volume of an oblique prism? Explain.
Share & Summarize

1. Write a formula for the volume of a prism based on the area of the base $A$ and the height $h$. Explain why your formula works.

2. Write a formula for the volume of a rectangular prism, using $l$, $w$, and $h$. Write another formula for the volume of a cylinder, using $r$ and $h$.

### Modeling with Block Structures

Many adults find hot tubs, saunas, steam rooms, and hot baths relaxing. In public places, though, these are off-limits to babies and young children, because children are affected much more quickly by heat than adults are. Why do you think this is true?

Your body uses energy, in the form of calories taken from food you have eaten, to keep its temperature relatively constant. When your body grows too hot, you start to sweat. As the sweat evaporates, it carries heat away from your skin, and you feel cooler.

Sweat is composed mostly of water and minerals. The more you sweat, the more your body cools—and the more water it loses. When your body is hot—because of the weather, because you are active, or because you are relaxing in a hot bath—you need to drink fluids to replace the water you are losing through sweat. Otherwise, you can become dehydrated and very sick. Because you sweat only through your skin, how fast you cool off—and how fast you lose water—depends on how much skin you have. That is, it depends on your surface area.

### Just the Facts

Pigs don’t sweat. After they roll around in mud, the mud dries and cools them. So “sweating like a pig” means not sweating at all!
Think & Discuss

Build models of two different 8-block “animals.” Design them so that one would cool more quickly than the other.

Explain the reasoning you used to construct your animals.

Building animals from blocks may sound silly. After all, blocks are nothing like animals. However, these models actually work quite well.

When mathematicians build a model, they ignore less-important details in order to simplify the situation they are modeling. Using blocks, you built two shapes that represented the relative sizes of the animals. You used the same number of blocks for each animal so that your animals had the same volume. Then you were able to compare just their surface areas.

Problem Set E

Mathematical models of a human baby and a human adult can help you make more comparisons between surface area and volume. For these problems, use a single block to model a baby and a $2 \times 2 \times 2$ cube to model an adult.

1. Given that the baby is represented by 1 block, consider whether the model for the adult is a good model.
   a. If the baby weighs 15 lb, how much does the adult weigh? Does that seem about right?
   b. Are adults really twice as large as babies in every dimension (length, width, and height)?

2. What is the volume of the model of a baby? What is its surface area?

3. What is the volume of the model of an adult? What is its surface area?

4. For every 1 cubic unit of the adult’s volume, how many square units of surface area are there? Is this more or less than the surface area of the baby’s 1 cubic unit?
Problem Set F

As you do these problems, consider both Shaunda’s and Luis’s reasoning.

1. For each structure, answer this question: For every 1 cubic unit of volume, how many square units of surface area are there?
   a. 
   b. 
   c. 

2. Continue to use the single block as a model for a baby, but build a new block model for an adult that you think is better than a $2 \times 2 \times 2$ cube. (It does not have to be a cube, a prism, or any other special shape, but it should be about the right size and shape compared to the model of a baby.) Draw a top-count view of your model.

3. Explain why you think your model of an adult is a better model than a $2 \times 2 \times 2$ cube.

4. What is the volume of your adult?

5. What is the surface area of your adult?
6. For every block in your adult model, how many square units of surface area are there?

7. For its size, does your adult have relatively more or less surface area than the baby has?

**Share & Summarize**

1. It is not safe to leave a baby in a closed car on a hot day because it can quickly become dehydrated (lose water)—much more quickly than an adult would. Explain why this is true.

2. In chilly weather, babies get colder much more quickly than adults do, so they must be wrapped warmly. Explain why.

---

**Lab Investigation**

**The Soft Drink Promotion**

Bursting Bubbles soft drink company wants to attract attention to its products at an upcoming convention. The company normally packages its soft drinks in 350-milliliter cylindrical cans that are 15 centimeters high.

Bursting Bubbles wants to show creativity at the convention by packaging its soft drinks in new containers of different shapes. The company has commissioned you, a mathematician, to investigate some other sizes and shapes and to make a recommendation. Company representatives tell you that the new containers can be any height at all—but they must have a volume of 350 milliliters. It’s now up to you to create some attention-getting containers.

The company gave the required volume in milliliters (mL). You probably already know that there are 1,000 mL in 1 liter (L). To do this problem, you also need to know that 1 milliliter has the same volume as 1 cubic centimeter.
CHAPTER 2

Geometry in Three Dimensions

Remember
The number π, pronounced “pie” and spelled pi, is about 3.14. Your calculator probably has a π key to make these calculations easier.

Make a Prediction
You decide to design some cylindrical containers first. As the height of the can changes, the base area must also change to keep the volume fixed at 350 milliliters.

1. You could design a really tall container—even as tall as 1 m (100 cm)! To keep the volume 350 mL, would the radius of the base circle have to increase or decrease as the height of the can increased?

2. Suppose you designed a very short can—even as short as 2 cm. To keep the volume 350 mL, would the radius of the base circle have to increase or decrease as the height decreased?

Try It Out

3. Bursting Bubble’s standard cans are 15 cm high. What is the radius of the circular base of these cans? To find the radius, you can use a strategy of systematic trial and error. The table shows the volume for a base radius of 3 cm and of 2.5 cm. Use a calculator to find the radius that would give a volume of 350 mL. Keep searching until you get within 1 mL of 350 mL.

<table>
<thead>
<tr>
<th>Base Radius (cm)</th>
<th>Base Area (cm²) (A = π × r × r)</th>
<th>Volume (mL, want 350) (base area × height)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>28.27</td>
<td>424.05 mL (too large)</td>
</tr>
<tr>
<td>2.5</td>
<td>19.635</td>
<td>294.525 mL (too small)</td>
</tr>
</tbody>
</table>

4. Choose five heights from the table below and make your own table. Complete your table with radius values that give a volume within 1 mL of 350 mL.

<table>
<thead>
<tr>
<th>Height (cm)</th>
<th>Radius of Base Circle (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
</tr>
</tbody>
</table>

Height (cm) | Radius of Base Circle (cm) |
-------------|---------------------------|
30           |                            |
50           |                            |
75           |                            |
90           |                            |
100          |                            |
150          |                            |

5. Were your predictions from Questions 1 and 2 correct?
Try It Again

The committee wants you to consider other shapes, not just cylinders, that might get attention at the convention. You can think about cones, spheres, other prisms—anything you want!

Here are two shapes, with formulas for their volumes. Use this information to answer the next set of questions.

6. Try the cone shape first. Choose at least three heights for the cone. For each height, find the radius needed to give a cone a volume of 350 mL. Get within 1 mL of 350 mL.

7. Using the formula for the volume of a sphere, find as many spheres as you can with a volume of 350 mL.

8. Find at least two other shapes for containers with a volume of 350 mL.

What Did You Learn?
Choose one design to present to the company.

9. Describe your design to the company, including its dimensions.

10. Explain why your design might be successful at the convention.

11. Make a model of your design to show the company.
1. Consider all the rectangular prisms that can be made with 27 blocks.
   a. Give the dimensions of each prism.
   b. Which of your 27-block prisms has the greatest surface area? Which has the least surface area?

2. Think of a top view made of squares that is different from the top views you have seen so far.
   a. Draw your top view.
   b. Suppose you wanted to build a prism 2.5 units high with that top view. What would its volume be?
   c. Write an expression for the volume of the prism, using $h$ for height.

3. Here is a parallelogram.
   a. What is the area of the parallelogram?
   b. If you build a prism 1 cm high using this parallelogram as a base, what will its volume be?
   c. Draw two other bases for containers that would have the same volume for a 1-cm height.

4. Six equilateral triangles are joined to form a hexagon.
   a. What is the area of the hexagon? Explain how you found your answer.
   b. If you built a prism on this base with a height of $h$ cm, what would its volume be?
5. The base of a cylinder has radius $r$ meters.

\[ \text{a.} \quad \text{What is the area of the cylinder’s base?} \]
\[ \text{b.} \quad \text{If the height of the cylinder is 1 m, what is its volume (in m}^3)\text{?} \]
\[ \text{c.} \quad \text{If the height is 10 m, what is the volume?} \]
\[ \text{d.} \quad \text{What is the cylinder’s volume if its height is } h \text{ meters?} \]

6. You can find, or at least make a good estimate of, the volume of a room in your home.

\[ \text{a.} \quad \text{Draw the base (floor) of the room, and show the measurements in feet or meters. If you measured exactly, say so. If you estimated, describe how you made your estimates.} \]
\[ \text{b.} \quad \text{What is the height of the room? Do you know exactly, or did you approximate it?} \]
\[ \text{c.} \quad \text{What is the volume of the room?} \]

7. Give the dimensions of four different containers that each have a volume of 360 cubic centimeters. They should not all be rectangular prisms.

8. For each structure, answer this question: For each 1 cubic unit of volume, how many square units of surface area are there?

\[ \text{a.} \]
\[ \text{b.} \]
\[ \text{c.} \]
9. **World Cultures** Countries keep statistics on the average weights of newborns and adults.

   a. Babies born in the United States have an average weight of 7.25 lb. Adults in the United States have an average weight of 160 lb. How heavy is an adult compared to a newborn (in other words, how many times as heavy)?

   b. If a block is your model of a newborn, how many blocks would be in your model of an adult? How would you arrange them?

   c. Babies born in the United States have an average length of 20 in. Adults in the United States have an average height of 5 ft 7 in. How tall is an adult compared to a baby (how many times as tall)?

   d. Use the information from Part c to check your answer for Part b. Now how would you arrange the blocks?

10. Suppose you want to design five different covered boxes that each hold the same amount of sand. You want each box to hold exactly 500 cm$^3$.

   a. Describe five different containers with this volume. Draw a top-count view of each structure, or draw the base (including the dimensions) and tell how high each structure will be.

   b. Which of your boxes has the least surface area? Is that the least surface area possible for a 500-cm$^3$ box?
11. A standard playing card is about 5.7 cm wide and 8.9 cm long. A stack of 52 cards—a whole deck—is about 1.5 cm high.
   a. What is the volume of a deck of cards?
   b. Use your answer to Part a to find the volume of a single card.

12. Many cereal boxes have dimensions of about 6 cm by 20 cm for the base and are about 27 cm high.
   a. What volume of cereal could this shape box hold?
   b. Give the dimensions (in cm) of four other rectangular boxes that have this same volume.
   c. Give the dimensions of two other containers that have this same volume. They do not have to be rectangular boxes.

13. Here are three cylinders that you probably have in your house. Pick one and find its volume.
   • a penny (It is hard to measure the height of a single penny, but it’s not hard to measure the height of 10 pennies and then divide.)
   • a soup can
   • a strand of spaghetti

14. Which of these containers holds the most water?

15. Draw two glasses that look different but hold the same amount of water. Show their dimensions.
16. Tino has two Alaskan malamutes, Trooper and Scooter. Scooter is Trooper’s puppy.
   a. Does Trooper have more or less total surface area than Scooter?
   b. Does Trooper have more or less surface area for a given unit of volume than Scooter?
   c. Which dog will cool off more quickly, Trooper or Scooter?

17. Life Science One of the plants pictured here lives in the desert and must conserve water. The other plant lives in the rain forest and does not need to conserve water. Describe how surface area and volume relationships could help you determine which plant is which.

Find each fractional amount.
18. \(\frac{3}{4}\) of 12
19. \(\frac{1}{2}\) of 50
20. \(\frac{3}{5}\) of 15
21. \(\frac{2}{3}\) of 12
22. \(\frac{4}{5}\) of 100
23. \(\frac{5}{4}\) of 100

Complete each table.

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<th>Percent</th>
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25. | Fraction | Decimal | Percent |
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<tr>
<td>(\frac{1}{8})</td>
<td>0.125</td>
<td></td>
</tr>
</tbody>
</table>
26. If there are \( n \) blocks in each bag, write an expression for the total number of blocks.

27. If there are \( n \) blocks in each bag, write an expression for the total number of blocks.

28. Complete the table.

\[
\begin{array}{c|ccccc}
 n & 0 & 1 & 2 & 3 & 10 & 50 \\
 n + 9 & & & & & & 110 \\
\end{array}
\]

29. The formula for the perimeter of a rectangle is \( 2L + 2W \), where \( L \) is the length of the rectangle and \( W \) is the width. Find the perimeter of a rectangle with length 6 cm and width 2 cm.

30. This table is from a game of *What's My Rule?* Find two ways of writing the rule in symbols. Choose your own variable.

\[
\begin{array}{c|ccccc}
 \text{In} & 0 & 1 & 2 & 3 & 4 \\
 \text{Out} & 3 & 6 & 9 & 12 & 15 \\
\end{array}
\]

31. **Probability** On a scale from 0 (impossible) to 100 (very likely), rate the chances of each event happening today.

   a. You do homework.
   
   b. Someone in your state has a baby.
   
   c. Someone in your city has a baby.
   
   d. You learn to drive.
   
   e. It snows somewhere in your country.
32. Zoe wants to build a doghouse for her dogs Trooper and Scooter. She made this sketch to work from.

\[ \text{3 ft} \]
\[ \text{2.6 ft} \]
\[ \text{5 ft} \]
\[ \text{3 ft} \]
\[ \text{3 ft} \]

\( a. \) Zoe plans to make the doghouse out of plywood, and she wants to include a floor. How many square feet of plywood will she need?

\( b. \) The hardware store sells scrap plywood for 25¢ a square foot, in increments of 10 square feet. How much will Zoe spend on plywood?

\( c. \) Zoe plans to cut a circular door in the front of the doghouse, and then paint the outside of the house (but not the floor). If the door has a diameter of 2 ft, how much area will Zoe need to paint to give the house two coats?

\( d. \) If a small can of paint covers 30 square yards, how many cans will Zoe need to buy?

\( e. \) The hardware store sells discount carpeting for $3 per square yard. How much will it cost Zoe to buy carpet for the floor of her doghouse?
A **net** is a flat figure that can be folded to form a closed, three-dimensional object. Such an object is called a **solid**.

**Explore**

- Which of these figures are also nets for a cube? That is, which will fold into a cube? Cut out a copy of each figure and try to fold it into a cube.

- Find and draw three other nets for a cube.

With your class, compile all the different nets for a cube that were found. How many nets are there? Do you think your class found all of them?
Investigation 1  Will It Fold?

For a net to form a closed solid, certain lengths have to match. For example, in the nets for cubes, the side lengths of all the squares must be the same. You will now investigate whether other nets will form closed solids. Pay close attention to the measurements that need to match.

Problem Set A

Decide whether each figure is a net—that is, whether it will fold into a solid. You might be able to decide just by looking at the figure. You can also cut out a copy of the figure and try to fold it. If the figure is a net, describe the shape it creates. If it isn’t a net, tell what goes wrong.

1.  

2.  

3.  

4.  

5.  

6.  

MATERIALS

scissors
Investigation 2 Using Nets to Investigate Solids

You have calculated surface area by counting the squares on the outside of a block structure. If the faces of a solid are not squares—like the figures below—you can find the solid’s surface area by adding the areas of all its faces.
Each net shown will fold to form a closed solid. Use the net’s measurements to find the surface area and volume of the solid. If you can’t find the exact volume, approximate it. It may be helpful to cut out and fold a copy of the net.

1.

2.

Remember
The formula for the circumference of a circle is $2\pi r$, where $r$ is the radius; or $\pi d$, where $d$ is the diameter.

EXAMPLE
This net folds to form a cylinder.

Because the net folds into a cylinder, the circles must be the same size. The area of each circle is $\pi r^2$ cm$^2$, or $25\pi$ cm$^2$. This is about 78.5 cm$^2$.

The length of the rectangle must be equal to the circumference of each circle, which is $10\pi$ cm, or about 31.4 cm. So the area of the rectangle is about 314 cm$^2$.

To find the cylinder’s surface area, just add the three areas:

$$78.5 \text{ cm}^2 + 78.5 \text{ cm}^2 + 314 \text{ cm}^2 = 471 \text{ cm}^2$$
Remember

The formula for the area of a triangle is \( \frac{1}{2}bh \), where \( b \) is the base and \( h \) is the height.

3.

4.

5.
Investigation 3 Is Today’s Soft Drink Can the Best Shape?

Bursting Bubbles soft drink company is trying to reduce their manufacturing costs in order to increase their profits. The president of the company wonders if there is a way to use less material to make a soft drink can.

How could Bursting Bubbles reduce the amount of aluminum it takes to make a can? One way is to design a can that has the minimum surface area for the volume of beverage it contains.

Problem Set C

To get started, inspect a 12-oz soft drink can. Get an idea of its dimensions, volume, and surface area.

1. Find the area in square centimeters of the base of the can. (If the can you are using is indented at the bottom, assume that it isn’t.)

2. Measure the can’s height in centimeters.
The volume printed on the can is given in milliliters (abbreviated mL). A milliliter has the same volume as a cubic centimeter, so \(1 \text{ mL} = 1 \text{ cm}^3\).

3. Does multiplying the area of the base by the height give the volume that is printed on the can? If not, why might the measures differ?

4. What is the surface area of the can in square centimeters? How did you find it? (Hint: Imagine what a net for the can would look like.)

**Problem Set**

Now consider how the can design might be changed. For these problems, use the volume you calculated in Problem Set C, rather than the volume printed on the can.

1. Design and describe five other cans that have approximately the same volume as your can. Sketch a net for each can, and record the radius of the base and the height. Include cans that are both shorter and taller than a regular soft drink can. (Hint: First choose the height of the can, and then find the radius.)

2. Calculate the surface area of each can you designed. Record your group’s data on the board for the class to see.

3. Compare the surface areas of the cans you and your classmates found. Which can has the greatest surface area? Which has the least surface area?

Bursting Bubbles wants to make a can using the least amount of aluminum possible. They can’t do it by evaluating every possible can, like the five you found, and choosing the best. There are too many—an infinite number!—and they could never be sure they had found the one using the least material.

One problem-solving strategy that mathematicians use is to gather data, as you did in Problem 1, and look for patterns in the data. For a problem like this one, they would also try to show that a particular solution is the best in a way that doesn’t involve testing every case.

4. Use what you know about volume and any patterns you found in Problems 1 and 2 to recommend a can with the least possible surface area to the president of Bursting Bubbles. Describe your can completely. Explain why you believe it has the least surface area.

**Share & Summarize**

Does the shape of standard soft drink cans use the minimum surface area for the volume they contain? If not, what might be some reasons companies use the shape they do?
On Your Own Exercises

Decide whether each figure is a net—that is, whether it will fold into a closed solid. One way to decide is to cut out a copy of the figure and try to fold it. If the figure is a net, describe the shape it creates. If it isn’t a net, tell what goes wrong.

1. 

2. 

3. 

4. A square pyramid has a square base and triangular faces that meet at a vertex. Find and draw at least three nets for a square pyramid.

In Exercises 5–8, find the surface area of the solid that can be created from the net. Show how you found the surface area.

5. Each triangle is equilateral, with sides 5 cm and height 4.3 cm.
6. Each triangle is equilateral, with sides 8 cm and height 7 cm. The other shapes are squares.

7. The radius of each circle is 2 cm. The height of the parallelogram is 2 cm.

8. The pentagons have side lengths of 3 cm. If you divide each pentagon into five isosceles triangles, the height of each triangle is 2 cm. The other shapes are squares.

9. An unopened box of tissues has length 24 cm, width 12 cm, and height 10.5 cm.
   a. What is the volume of the box?
   b. What is the surface area of the box?

10. A juice pitcher is shaped like a cylinder. It is 30 cm tall and has a base with radius 6 cm. It has a flat lid.
    a. How much juice will the pitcher hold?
    b. What is the surface area of the pitcher?
11. **Challenge** This net will fold into a *triangular pyramid*. Find the surface area of the pyramid, and estimate its volume. You may want to fold a copy of the net into the pyramid.

![Triangular Pyramid Net]

12. Choose a container in your house. You might choose a can (but not a soft drink can), a rectangular box, or some other object.
   - **a.** Draw a net for the object.
   - **b.** Find the surface area and volume of the object by measuring its dimensions.
   - **c.** Draw nets for three other objects with the same volume as your container, and find the surface area of each. The new objects do not have to be real containers from your home.
   - **d.** Is there a prism or cylinder with less surface area but the same volume as your container? If so, draw a net for it.

13. Imagine removing a label from a soup can.
   - **a.** What is the shape of the label?
   - **b.** If the radius of the soup can is \( r \) and the height is \( h \), what are the dimensions of the label?

Create a net for each box.

14. ![Net 1]
15. ![Net 2]
16. **Challenge**

17. **Challenge** A cone has a circular base and a vertex some height away from that base. Make a net for a cone.
18. **Challenge** A *tetrahedron* is a solid with four triangular faces. You have made tetrahedrons from nets in this lesson. Nets for a tetrahedron contain only triangles. Can you make a net for another figure using only triangles? (Hint: You might try taping paper triangles together to form a new solid.)

19. A circular pond is 100 ft in diameter. In the middle of winter, the ice on the pond is 6 inches thick.
   a. Think of the ice on the pond as a cylinder. What is the volume of ice?
   b. What is the surface area of the floating cylinder of ice?
   c. What would be the edge length of a cube of ice that had the same volume?
   d. What would be the surface area of a cube of ice that had the same volume?
   e. Which has greater surface area: the cylinder of ice or the cube of ice? How much more?

20. Kinu’s Ice Cream Parlor packs ice cream into cylindrical tubs that are 20 cm in diameter and 30 cm tall.
   a. How much ice cream does a tub hold?
   b. Kinu’s Ice Cream Parlor sells ice cream in cylindrical scoops. If each scoop has base radius 2 cm and height 6 cm, what is the volume of a single scoop?
   c. How many scoops of ice cream are in a tub? Show how you found your answer.
   d. What is the surface area of a tub?
   e. What is the total surface area of the ice cream from one tub *after* it has all been scooped out?
   f. Which has more surface area: the ice cream in the tub or the scooped ice cream? Which would melt faster?
Mixed Review

Evaluate each expression.

21. \(0.6 \cdot 0.6\)  \hspace{1cm} 22. \(0.3 \cdot 0.3\)  \hspace{1cm} 23. \(0.02 \cdot 0.02\)

24. \(0.8 + 0.01\)  \hspace{1cm} 25. \(0.8 - 0.01\)  \hspace{1cm} 26. \(2 - 0.8\)

27. Complete the table.

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</tr>
</tbody>
</table>

Write each expression without using multiplication signs.

28. \(b \times b \times b \times b\)  \hspace{1cm} 29. \(y \times y \times y \times 3\)  \hspace{1cm} 30. \(a \times a \times a \times a \times 4\)

Find the value for \(2k^2\) for each value of \(k\).

31. \(k = 0\)  \hspace{1cm} 32. \(k = \frac{1}{2}\)  \hspace{1cm} 33. \(k = 1.2\)

Use the given expression to complete each table. Then write another expression that gives the same values.

34. \(t\)  \hspace{1cm} 35. \(r\)

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</table>

<table>
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</table>

36. Ecology  The table lists the number of endangered species for five groups of animals.

a. Make a circle graph of these data. On each section, write the percentage each category is of the total number of species listed. Round to the nearest tenth.

b. About what percentage of the endangered species in the five groups are mammals or birds?

c. Write three statements comparing the number of endangered fish species to the number of endangered snail species.
Chapter Summary

In this chapter, you have explored four representations of three-dimensional figures:

- top-count views
- engineering views
- regular views (front, top, and right)
- nets

You learned that volume is measured in cubic units, and that area and surface area are measured in square units. You found volume and surface area for block structures by counting cubes and counting exposed faces. You also found a formula for the volume of prisms, and a method for finding surface area by using nets.

Finally, you investigated a scientific application of surface area and volume: the relationship between surface area and volume in human beings, and what it reveals about cooling and dehydration.

Strategies and Applications

The questions in this section will help you review and apply the important ideas and strategies developed in this chapter.

Working with block patterns

1. Create your own block pattern.
   a. Draw top-count views for the first three stages of your pattern.
   b. Describe how you arrange blocks as you build from one stage of your pattern to the next. Your description should be detailed enough that someone could use it to build any stage from the previous one.
   c. Describe the number of blocks added from one stage of your pattern to the next. Write an expression for the number of blocks you add to Stage $s$ to make Stage $s + 1$.
   d. Write an expression for the total number of blocks in Stage $s$.

Representing three-dimensional structures

2. Use 10 blocks to create a structure.
   a. Draw the top-count view for your structure.
   b. Draw regular front, top, and right views of your structure.
   c. Draw engineering views for your structure.

3. Draw a net for the solid shown at left.
Finding the volume of a solid

4. A block prism has a base 4 cm long and 3 cm wide. The prism is 5 cm tall.
   a. What is the prism’s volume?
   b. Imagine a different block prism with the same height and volume as the original prism. What are its dimensions?
   c. Imagine a cylinder with height 5 cm and the same volume as the original prism. What is the area of its base? Estimate its radius.

Finding the surface area of a solid

5. Find all the rectangular prisms you can make with 24 blocks. Try to do this without building all of them.
   a. Give the dimensions of each prism.
   b. Which of your prisms has the greatest surface area? What is its surface area?
   c. Which of your prisms has the least surface area? What is its surface area?

Find the surface area of each solid.

6. 7. 8.

Remember
An isosceles triangle has two sides of equal length.

8. The square in the figure at the right has sides 3 cm long. The triangles are isosceles, and the height of each triangle is 4 cm. Is the figure a net? In other words, does it fold up to form a closed solid? Explain how you know.
Demonstrating Skills

In Questions 9–16, find the volume and surface area of the solid.

9. a cylindrical storage tank with radius 5 ft and height 12 ft
10. a cardboard box with length 2 m, width 2.5 m, and height 3 m
11. a can with height 10 cm and circumference 18 cm
12. a cube with side length 1.5 m

13. Which of these three solids has the greatest surface area? Which has the least surface area?

Remember

The circumference of a circle is \(2\pi r\), where \(r\) is the radius.